1 (a) It has often been said, to avoid relegation from football's Premier League, teams should aim to score at least 40 points in a season. Each team plays 38 games, a win is rewarded with 3 points and a draw with 1 point. No points are awarded for a loss.

Using W and D as the number of wins and draws respectively, write down two inequalities. One relating to the number of points a team needs to avoid relegation and one relating to the number of games played.

(2 marks)

(b) Explain why $W \geq 0$ and $D \geq 0$ must also be conditions related to the problem.

(1 mark)

(c) A team has won 4 games and drawn 4 after playing 17 games. Write down two updated inequalities for the number of wins and draws required for the remainder of the season in order to avoid relegation.

(2 marks)

2 (a) The leakage rate of water from a pipe, $L \text{ l s}^{-1}$ (litres per second), is directly proportional to the square root of the flow rate, $s \text{ m s}^{-1}$ (meters per second), which is the speed of the water flowing through the pipe. It was observed that the leaking rate was 0.72 l s^{-1} when the flow rate was 0.64 m s^{-1}

Write down an equation connecting L and S.

(2 marks)

(b) Find the flow rate when the leakage rate is 0.49 l s^{-1}.

(2 marks)

(c) An alternative model for the leakage rate is $L = 0.5s$. Apart from when there is no leak find a flow rate and a leakage rate for when both models predict the same result.

(2 marks)

3 (a) A soft ball is thrown upwards from the top of a building. The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = 15 + 8.4t - 4.9t^2 \qquad t > 0$$

What is the significance of the constant 15 in the function?

(1 mark)

(b) At what time is the ball at the same height as when it was thrown?

(2 marks)

(c) Find the time at which the ball is at its maximum height and what this maximum height is.

(3 marks)

(d) How long does it take for the ball to first hit the ground?

(2 marks)

(e) Given that the ball first hits the ground at a distance 20 m from the base of the building find the shortest distance between this point and where the ball was thrown from.

(2 marks)

4 Write the following in the form ,e^{kx} where k is a constant and $k > 0$.

(i) $e^{3x} \times e^{2x}$

(ii) 5^x

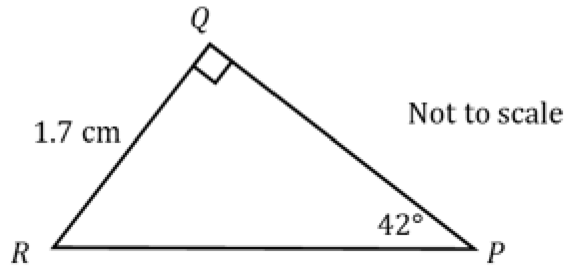
(iii) 2^x

(3 marks)

5 Sketch the graph of $y = x^2 - 1$, labelling any points where the graph intersects the coordinate axes.

(3 marks)

6 Find the length of the side *PQ* in the triangle *PQR* below, giving your answer to one decimal place.



(2 marks)

7 Find the coordinates of the midpoint of the straight line connecting the following points:

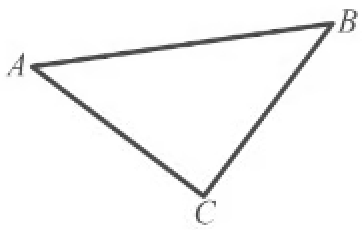
(i) (2 , 4) and (6 , 10),

(ii) (-3 , 6) and (5 , 9),

(iii) (0 , -8) and (3 , 2).

(5 marks)

8 In triangle ABC, $\overrightarrow{AB} = 5\mathbf{i} + \mathbf{j}$ *and* $\overrightarrow{AC} = 3\mathbf{i} - 2\mathbf{j}$.



(i) Find $\overrightarrow{BC}$ in terms of **i** and **j**.

(ii) Calculate $\left|\overrightarrow{BC}\right|$.

(4 marks)

9 (a) Show that the equation $4x + 2y - 6 = 0$ can be written as $y = 3 - 2x$.

(1 mark)

(b) By substituting the result from part (a) into the equation $3x+2y-1=1$ solve the equations

$$4x+2y-6=0$$
$$3x+2y-1=1$$

(3 marks)

10 Find the length of the straight line segments connecting the following points:

(i) (2 , 4) and (5 , 8),

(ii) (3 , - 6) and (-2, -14),

(iii) (5, -13) and (2, -7).

(5 marks)

11 On separate diagrams sketch the circles with the following equations

(i) $x^2+y^2=9$

(ii) $(x-4)^2+(y-3)^2=4^2$

(4 marks)

12 Complete the square for

(i) x^2+8x-4

(ii) $2x^2+12x-5$

(iii) $5x^2-3x+2$

(3 marks)

13 Expand and simplify

(i) $(2x+3)(x-4)$

(ii) $2p(p+3)(p-2)$

(iii) $(y-1)(y-2)^2$

(5 marks)

14 Solve the inequalities:

(i) $2x \geq 8$

(ii) $3 + 2x < 11$

(iii) $5 + x > 4x - 1$

(3 marks)

15 Use the factor theorem to verify that $(x - 2)$ is a factor of $x^3 - x^2 - 14x + 24$.

(2 marks)

16 The equation $x^2 + kx + 4 = 0$, where k is a constant, has no real roots.

Find the possible value(s) of k.

(4 marks)

17 (a) Given that $(x + 1)$ is a factor of $\mathrm{f}(x) = x^3 - 5x^2 + 3x + 9$, fully factorise $\mathrm{f}(x)$.

(4 marks)

(b) Sketch the graph of $y = \mathrm{f}(x)$, labelling the coordinates of all points where the graph intersects the coordinate axes.

(3 marks)

18 The vectors **a** and **b** are given by $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j}$ and $\mathbf{b} = -\mathbf{i} + 3\mathbf{j}$.

Find:

(i) **a + b**

(ii) 5**a,**

(iii) 3**a - **2**b**

(iv) **a -** *t* **b,**

(5 marks)

19 (a) Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle, where $\mathbf{F}_1 = 7\mathbf{i} - 2\mathbf{j}$ newtons and $\mathbf{F}_2 = -12\mathbf{i} - 10\mathbf{j}$ newtons.

The resultant force **R** acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$.

Calculate the magnitude of **R** in newtons.

(3 marks)

(b) A third force $\mathbf{F}_3 = k\mathbf{j}$ newtons is to be applied to the particle. The constant k is to be selected so that the line of action of the new resultant force $\mathbf{R}_{new} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ is at an angle of 45 degrees to the vector **j**, measured anticlockwise.

Find the value of k.

(3 marks)

20 (a) In an experiment, three forces are acting on a particle. $\mathbf{F}_1 = 7\mathbf{i} - \mathbf{j}$ newtons and $\mathbf{F}_2 = x\mathbf{i} + y\mathbf{j}$ newtons are both constant forces, although the values of x and y are initially unknown. The third force is $\mathbf{F}_3 = k\mathbf{i} + k\sqrt{3}\mathbf{j}$ newtons, where $k \geq 0$ is a parameter that can be varied by the experimenters. The resultant force $\mathbf{R}$ acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$.

Given that $\mathbf{R} = \mathbf{0}$ when the magnitude of $\mathbf{F}_3$ is 10 newtons, find the exact values of x and y.

(4 marks)

(b) Find the magnitude of $\mathbf{F}_2$ and the angle it makes with the vector $\mathbf{i}$. Give your answers correct to 1 decimal place.

(3 marks)

Answers

It has often been said, to avoid relegation from football's Premier League, teams should aim to score at least 40 points in a season. Each team plays 38 games, a win is rewarded with 3 points and a draw with 1 point. No points are awarded for a loss.

(a) Using W and D as the number of wins and draws respectively, write down two inequalities. One relating to the number of points a team needs to avoid relegation and one relating to the number of games played.

[2]

(b) Explain why $W \geq 0$ and $D \geq 0$ must also be conditions related to the problem.

[1]

(c) A team has won 4 games and drawn 4 after playing 17 games. Write down two updated inequalities for the number of wins and draws required for the remainder of the season in order to avoid relegation.

[2]

a)

AVOID RELEGATION

$3w + D \geqslant 40$

NUMBER OF GAMES

$W + D \leqslant 38$

$\boxed{3w + D \geqslant 40 \qquad W + D \leqslant 38}$

1 (a) **(2 marks)**

It has often been said, to avoid relegation from football's Premier League, teams should aim to score at least 40 points in a season. Each team plays 38 games, a win is rewarded with 3 points and a draw with 1 point. No points are awarded for a loss.

(a) Using W and D as the number of wins and draws respectively, write down two inequalities. One relating to the number of points a team needs to avoid relegation and one relating to the number of games played.

[2]

(b) Explain why $W \geq 0$ and $D \geq 0$ must also be conditions related to the problem.

[1]

(c) A team has won 4 games and drawn 4 after playing 17 games. Write down two updated inequalities for the number of wins and draws required for the remainder of the season in order to avoid relegation.

[2]

b)

NUMBER OF GAMES CANNOT BE NEGATIVE

(b) **(1 mark)**

It has often been said, to avoid relegation from football's Premier League, teams should aim to score at least 40 points in a season. Each team plays 38 games, a win is rewarded with 3 points and a draw with 1 point. No points are awarded for a loss.

(a) Using W and D as the number of wins and draws respectively, write down two inequalities. One relating to the number of points a team needs to avoid relegation and one relating to the number of games played.

[2]

(b) Explain why $W \geq 0$ and $D \geq 0$ must also be conditions related to the problem.

[1]

(c) A team has won 4 games and drawn 4 after playing 17 games.
Write down two updated inequalities for the number of wins and draws required for the remainder of the season in order to avoid relegation.

[2]

c) USING INEQUALITIES FROM (a)

① $3W + D \geq 40$ AND ② $W + D \leq 38$

ADD IN POINTS ALREADY ACHIEVED

① $3\times4 + 3W + 4 + D \geq 40$

$16 + 3W + D \geq 40$

$-16 \qquad -16$

$3W + D \geq 24$

ADD IN GAMES ALREADY PLAYED

② $17 + W + D \leq 38$

$-17 \qquad -17$

$W + D \leq 21$

$\boxed{3W + D \geq 24 \qquad W + D \leq 21}$

(c) **(2 marks)**

The leakage rate of water from a pipe, L l s^{-1} (litres per second), is directly proportional to the square root of the flow rate, s m s^{-1} (meters per second) which is the speed of the water flowing through the pipe.
It was observed that the leaking rate was 0.72 l s^{-1} when the flow rate was 0.64 m s^{-1}.

(a) Write down an equation connecting L and s.

[2]

(b) Find the flow rate when the leakage rate is 0.49 l s^{-1}.

[2]

(c) An alternative model for the leakage rate is $L = 0.5s$.
Apart from when there is no leak find a flow rate and a leakage rate for when both models predict the same result.

[2]

a)

CREATE PROPORTIONALITY EQUATION

$L \propto \sqrt{s}$ $\qquad L = k\sqrt{s}$

SOLVE FOR K

$0.72 = k\sqrt{0.64}$

$k = \frac{0.72}{0.8} = 0.9$

SUB INTO EQUATION

$\boxed{L = 0.9\sqrt{s}}$

2 (a) **(2 marks)**

The leakage rate of water from a pipe, L l s^{-1} (litres per second), is directly proportional to the square root of the flow rate, s m s^{-1} (meters per second) which is the speed of the water flowing through the pipe.
It was observed that the leaking rate was 0.72 l s^{-1} when the flow rate was 0.64 m s^{-1}.

(a) Write down an equation connecting L and s.

[2]

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[2]

(c) An alternative model for the leakage rate is $L = 0.5s$.
Apart from when there is no leak find a flow rate and a leakage rate for when both models predict the same result.

[2]

b)

$L = 0.9\sqrt{s}$

SUB IN $L = 0.49$

$0.49 = 0.9\sqrt{s}$

$\sqrt{s} = \frac{0.49}{0.9}$

$s = \left(\frac{0.49}{0.9}\right)^2 = 0.2964...$

$\boxed{0.296 \text{ m s}^{-1}}$

(b) **(2 marks)**

The leakage rate of water from a pipe, L l s^{-1} (litres per second), is directly proportional to the square root of the flow rate, s m s^{-1} (meters per second) which is the speed of the water flowing through the pipe.
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[2]

(c) An alternative model for the leakage rate is $L = 0.5s$.
Apart from when there is no leak find a flow rate and a leakage rate for when both models predict the same result.

[2]

c) SIMULTANEOUS EQUATIONS

① $L = 0.5s$ ② $L = 0.9\sqrt{s}$

$0.5s = 0.9\sqrt{s}$

$0.5s - 0.9\sqrt{s} = 0$

HIDDEN QUADRATIC $f(x) = \sqrt{s}$

$0.5x^2 - 0.9x = 0$

$x(0.5x - 0.9) = 0$

$\cancel{x = 0}$ $x = \frac{0.9}{0.5} = 1.8$

$f(x) = \sqrt{s}$

$x = \sqrt{s}$ $\sqrt{s} = 1.8$

$s = 1.8^2 = 3.24 \text{ ms}^{-1}$

SUB INTO ①

$L = 0.5 \times 3.24 = 1.62 \text{ Ls}^{-1}$

CHECK IN ②

$1.62 = 0.9\sqrt{3.24}$ ✓✓

$$\boxed{s = 3.24 \text{ ms}^{-1} \quad L = 1.62 \text{ Ls}^{-1}}$$

(c) **(2 marks)**

A soft ball is thrown upwards from the top of a building.
The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = 15 + 8.4t - 4.9t^2 \qquad t > 0$$

(a) What is the significance of the constant 15 in the function? [1]

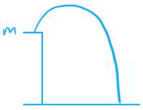
(b) At what time is the ball at the same height as when it was thrown? [2]

(c) Find the time at which the ball is at its maximum height and what this maximum height is. [3]

(d) How long does it take for the ball to first hit the ground? [2]

(e) Given that the ball first hits the ground at a distance 20 m from the base of the building find the shortest distance between this point and where the ball was thrown from. [2]

a) SKETCH MODEL TO VISUALISE PROBLEM



$\boxed{\text{15m = HEIGHT OF BUILDING OR STARTING POINT OF BALL}}$

3 (a) **(1 mark)**

A soft ball is thrown upwards from the top of a building.
The height, h m of the ball above the ground after t seconds is modelled by the function

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(e) Given that the ball first hits the ground at a distance 20 m from the base of the building find the shortest distance between this point and where the ball was thrown from. [2]

b) SOLVE FOR $h(t) = 15$

$$\cancel{15} = \cancel{15} + 8.4t - 4.9t^2$$

$$4.9t^2 - 8.4t = 0$$

$$t(4.9t - 8.4) = 0$$

$t = 0 \qquad t = \frac{8.4}{4.9} = 1.714\ldots$

$\boxed{t = 1.71 \text{ SECONDS}}$

(b) **(2 marks)**

A soft ball is thrown upwards from the top of a building.
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(e) Given that the ball first hits the ground at a distance 20 m from the base of the building find the shortest distance between this point and where the ball was thrown from. [2]

c) COMPLETE THE SQUARE $\quad a(x-b)^2+c$

MAX $(+b, +c)$

$-4.9t^2 + 8.4t + 15 = 0$

$-4.9\left[t^2 - \frac{12}{7}t\right] + 15 = 0$

$-4.9\left[\left(t - \frac{6}{7}\right)^2 - \frac{36}{49}\right] + 15 = 0$

$-4.9\left(t - \frac{6}{7}\right)^2 + \frac{18}{5} + 15 = 0$

$-4.9\left(t - \frac{6}{7}\right)^2 + \frac{93}{5} = 0$

MAXIMUM

$(t, h) \qquad \left(\frac{6}{7}, \frac{93}{5}\right)$

$\boxed{t = 0.857 \text{ SECONDS} \quad h = 18.6\text{m}}$

(c) **(3 marks)**

A soft ball is thrown upwards from the top of a building.
The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = 15 + 8.4t - 4.9t^2 \qquad t > 0$$

(a) What is the significance of the constant 15 in the function? [1]

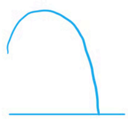
(b) At what time is the ball at the same height as when it was thrown? [2]

(c) Find the time at which the ball is at its maximum height and what this maximum height is. [3]

(d) How long does it take for the ball to first hit the ground? [2]

(e) Given that the ball first hits the ground at a distance 20 m from the base of the building find the shortest distance between this point and where the ball was thrown from. [2]

d) SOLVE FOR $h(t) = 0$



$4.9t^2 - 8.4t - 15 = 0$

$t = 2.8054...$

$t = -1.0911...$

IGNORE NEGATIVE SOLUTION

$\boxed{t = 2.81 \text{ SECONDS}}$

(d) **(2 marks)**

A soft ball is thrown upwards from the top of a building.
The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = 15 + 8.4t - 4.9t^2 \qquad t > 0$$

(a) What is the significance of the constant 15 in the function? [1]

(b) At what time is the ball at the same height as when it was thrown? [2]

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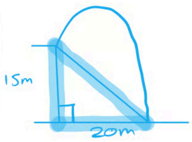
(e) Given that the ball first hits the ground at a distance 20 m from the base of the building find the shortest distance between this point and where the ball was thrown from. [2]

e) SKETCH MODEL TO VISUALISE PROBLEM



PYTHAGORAS

$a^2 + b^2 = c^2$

$c^2 = 15^2 + 20^2$

$225 + 400 = 625$

$c = \sqrt{625} = 25\text{m}$

$\boxed{25\text{m}}$

(e) **(2 marks)**

Write the following in the form e^{kx}, where k is a constant and $k > 0$.

(i) $e^{3x} \times e^{2x}$

(ii) 5^x

(iii) 2^x

[3]

i) $e^{3x} \times e^{2x} = e^{3x+2x}$ (Using power laws!)

$= \boxed{e^{5x}}$

$a = e^{\ln a}$ (since ln is the inverse of e)

ii) Let $a = 5^x$

$5^x = e^{\ln 5^x}$

$= \boxed{e^{(\ln 5)x}}$

iii) Let $a = 2^x$

$2^x = e^{\ln 2^x}$

$= \boxed{e^{(\ln 2)x}}$

4 **(3 marks)**

Sketch the graph of $y = x^2 - 1$, labelling any points where the graph intersects the coordinate axes.

QUADRATIC ∪

[3]

x INTERCEPTS SOLVE $y = 0$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm\sqrt{1} = \pm 1$$

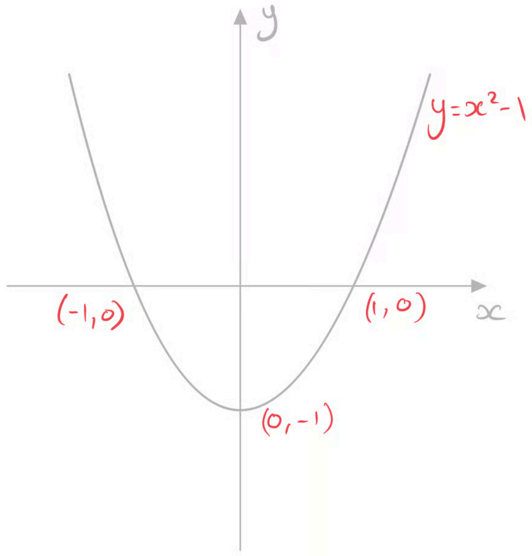
$$\boxed{(-1, 0) \quad (1, 0)}$$

y INTERCEPT $x = 0$

$$0^2 - 1 = -1$$

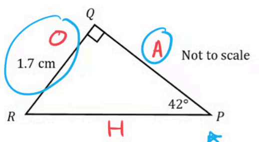
$$\boxed{(0, -1)}$$

DRAW CURVE THEN ADD AXES



5 **(3 marks)**

Find the length of the side PQ in the triangle PQR below, giving your answer to one decimal place.



[2]

LABEL TRIANGLE

(HYPOTENUSE, ADJACENT, OPPOSITE)

IDENTIFY WHICH SIDES ARE NEEDED (A, O)

PICK APPROPRIATE TRIG RATIO

$$S\,{}^{O}H \quad C\,{}^{A}H \quad T\,{}^{O}A$$

SUBSTITUTE VALUES AND REARRANGE

$$\tan x = \frac{O}{A} \qquad \tan 42° = \frac{1.7}{A}$$

$$A = \frac{1.7}{\tan 42} = 1.888\ldots$$

$$\boxed{PQ = 1.9 \text{ cm}}$$

6 **(2 marks)**

Find the coordinates of the midpoint of the straight line connecting the following points:

(i) $(2, 4)$ and $(6, 10)$,
(ii) $(-3, 6)$ and $(5, 9)$,
(iii) $(0, -8)$ and $(3, 2)$.

[5]

MIDPOINT

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

i) $\left(\frac{2+6}{2}, \frac{4+10}{2}\right) = \boxed{(4, 7)}$

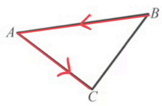
ii) $\left(\frac{-3+5}{2}, \frac{6+9}{2}\right) = \boxed{\left(1, \frac{15}{2}\right)}$

iii) $\left(\frac{0+3}{2}, \frac{-8+2}{2}\right) = \boxed{\left(\frac{3}{2}, -3\right)}$

ANSWERS CAN BE LEFT AS IMPROPER FRACTIONS WHERE APPROPRIATE

7 **(5 marks)**

In triangle ABC, $\overrightarrow{AB} = 5\mathbf{i} + \mathbf{j}$ and $\overrightarrow{AC} = 3\mathbf{i} - 2\mathbf{j}$.



THINK OF VECTORS AS JOURNEY

(i) Find $\overrightarrow{BC}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.
(ii) Calculate $|\overrightarrow{BC}|$.

[4]

MAGNITUDE OF VECTOR = LENGTH

i) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$ OR $\overrightarrow{AC} - \overrightarrow{AB}$

$3\underline{i} - 2\underline{j} - 5\underline{i} - \underline{j}$

$\boxed{\overrightarrow{BC} = -2\underline{i} - 3\underline{j}}$

ii) USING PYTHAGORAS

$|\overrightarrow{BC}| = \sqrt{(-2)^2 + (-3)^2}$ NEGATIVES CAN BE IGNORED

$= \sqrt{4+9}$

$\boxed{|\overrightarrow{BC}| = \sqrt{13}}$

LEAVE IN SURD FORM

8 **(4 marks)**

(a) Show that the equation $4x + 2y - 6 = 0$ can be written as $y = 3 - 2x$.

[1]

(b) By substituting the result from part (a) into the equation $3x + 2y - 1 = 1$ solve the equations

$4x + 2y - 6 = 0$
$3x + 2y - 1 = 1$

[3]

a)

$4x + 2y - 6 = 0$ $\quad +6-4x$

$2y = 6 - 4x$ $\quad \div 2$

$\boxed{y = 3 - 2x}$

9 (a) **(1 mark)**

(a) Show that the equation $4x + 2y - 6 = 0$ can be written as $y = 3 - 2x$.

[1]

(b) By substituting the result from part (a) into the equation $3x + 2y - 1 = 1$ solve the equations

$4x + 2y - 6 = 0$ ①
$3x + 2y - 1 = 1$ ②

[3]

b) Sub $y = 3-2x$ from (a) into ② to eliminate y

$3x + 2(3-2x) - 1 = 1$ expand brackets

$3x + 6 - 4x - 1 = 1$ simplify

$-x + 5 = 1$ $\quad +x, -1$

$4 = x$

Sub $x = 4$ into $y = 3-2x$ and solve to find y.

(We can also sub it into ① or ② but since y is the subject here, it will be faster to solve.)

$y = 3 - 2(4) = -5$

$\boxed{x = 4 \quad y = -5}$

(b) **(3 marks)**

Find the length of the straight line segments connecting the following points:

(i) (2 , 4) and (5 , 8),

(ii) (3 , −6) and (−2 , −14),

(iii) (5 , −13) and (2 , −7).

[5]

DISTANCE BETWEEN TWO POINTS

PYTHAGORAS

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

i) $\sqrt{(5-2)^2 + (8-4)^2}$

$\sqrt{3^2 + 4^2} = \sqrt{25} = \boxed{5}$

ii) $\sqrt{(-2-3)^2 + (-14 - -6)^2}$

BE CAREFUL WITH NEGATIVES

$\sqrt{(-5)^2 + (-8)^2} = \boxed{\sqrt{89}}$

LEAVE ANSWER IN SURD FORM UNLESS ASKED TO ROUND

iii) $\sqrt{(2-5)^2 + (-7 - -13)^2}$

$\sqrt{(-3)^2 + 6^2} = \sqrt{45}$

SIMPLIFY SURD IF POSSIBLE

$= \boxed{3\sqrt{5}}$

10 **(5 marks)**

On separate diagrams sketch the circles with the following equations

(i) $x^2 + y^2 = 9$

(ii) $(x-4)^2 + (y-3)^2 = 4^2$

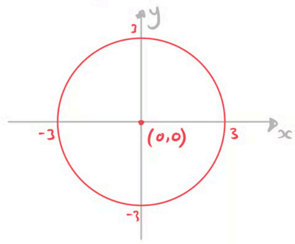
[4]

EQUATION OF A CIRCLE

$(x-a)^2 + (y-b)^2 = r^2$

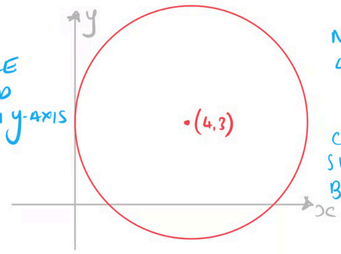
HAS CENTRE (a,b) RADIUS r

i) CENTRE (0,0) RADIUS = 3



ii) CENTRE (4,3) RADIUS = 4

CIRCLE SHOULD TOUCH y-AXIS



NO NEED TO LABEL AXES

CIRCLE SHOULD GO BELOW x-AXIS

11 **(4 marks)**

Complete the square for

(i) $x^2 + 8x - 4$
(ii) $2x^2 + 12x - 5$
(iii) $5x^2 - 3x + 2$

[3]

$y = ax^2 + bx + c$
$y = a(x+p)^2 + q$

i) $a=1,\ b=8,\ c=-4$

① $p = \frac{b}{2}$ $\quad p = \frac{8}{2} = 4$

② $q = c - p^2$ $\quad q = -4 - (4)^2 = -20$

③ $a(x+p)^2 + q$ $\quad \boxed{(x+4)^2 - 20}$

ii) $a \neq 1$

① Factor 'a' on RHS $\quad 2(x^2 + 6x) - 5$

② Working with (x^2+6x) only, $p = \frac{b}{2}$ $\quad p = \frac{6}{2} = 3$

③ Still working with (x^2+6x) only, $q = c - p^2$ $\quad q = 0 - 3^2 = -9$

④ Expand and simplify

$2((x+3)^2 - 9) - 5$
$= 2(x+3)^2 - 18 - 5$
$= \boxed{2(x+3)^2 - 23}$

iii) $a \neq 1$

① Factor 'a' on RHS $\quad 5(x^2 - \frac{3}{5}x) + 2$

② Working with $(x^2 - \frac{3}{5}x)$ only, $p = \frac{b}{2}$ $\quad p = -\frac{3}{5} \div 2 = \frac{-3}{10}$

③ Still working with $(x^2 - \frac{3}{5}x)$ only, $q = c - p^2$ $\quad q = 0 - (\frac{3}{10})^2 = \frac{-9}{100}$

④ Expand and simplify

$5((x - \frac{3}{10})^2 - \frac{9}{100}) + 2$
$= \boxed{5(x - \frac{3}{10})^2 + \frac{31}{20}}$

12 **(3 marks)**

Expand and simplify

(i) $(2x + 3)(x - 4)$
(ii) $2p(p + 3)(p - 2)$
(iii) $(y - 1)(y - 2)^2$

[5]

i) $(2x+3)(x-4) = 2x^2 - 8x + 3x - 12$ Expand
$= \boxed{2x^2 - 5x - 12}$ Simplify

ii) Multiply out the brackets first.

$(p+3)(p-2) = p^2 + 3p - 2p - 6$
$= p^2 + p - 6$

Then multiply the result by $2p$.

$2p(p^2 + p - 6) = \boxed{2p^3 + 2p^2 - 12p}$

iii) Multiply out 2 brackets first.

$(y-2)^2 = (y-2)(y-2) = y^2 - 2y - 2y + 4$
$= y^2 - 4y + 4$

Then multiply the result by the remaining pair of brackets.

$(y-1)(y^2 - 4y + 4) = y^3 - 4y^2 + 4y - y^2 + 4y - 4$
$= \boxed{y^3 - 5y^2 + 8y - 4}$

13 **(5 marks)**

Solve the inequalities:

(i) $2x \geq 8$

(ii) $3 + 2x < 11$

(iii) $5 + x > 4x - 1$

[3]

i) $2x \geqslant 8$

$\div 2$ $\div 2$

$\boxed{x \geqslant 4}$

Number Line

● $\leqslant\ \geqslant$

○ $<\ >$

ii) $3 + 2x < 11$

-3 -3

$2x < 8$

$\div 2$ $\div 2$

$\boxed{x < 4}$

iii) $5 + x > 4x - 1$

$-x+1$ $-x+1$

$6 > 3x$

$\div 3$ $\div 3$

$\boxed{2 > x}$

14 **(3 marks)**

Use the factor theorem to verify that $(x - 2)$ is a factor of $x^3 - x^2 - 14x + 24$.

[2]

Let $f(x) = x^3 - x^2 - 14x + 24$

Factor theorem: If $(x-2)$ is a factor of $f(x)$, then $f(2) = 0$.

$f(2) = (2)^3 - (2)^2 - 14(2) + 24 = 0$

So $(x-2)$ is a factor.

15 **(2 marks)**

The equation $x^2 + kx + 4 = 0$, where k is a constant, has no real roots.
Find the possible value(s) of k.

[4]

'no real roots' $\longrightarrow$ discriminant < 0

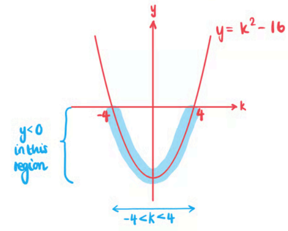
$a = 1 \quad b = k \quad c = 4$

$b^2 - 4ac = k^2 - 4(1)(4) = k^2 - 16$

$k^2 - 16 < 0$

$(k+4)(k-4) < 0$

Sketch the graph to see where the solutions are.



$\boxed{-4 < k < 4}$

16 **(4 marks)**

(a) Given that $(x+1)$ is a factor of $f(x) = x^3 - 5x^2 + 3x + 9$, fully factorise $f(x)$.

[4]

(b) Sketch the graph of $y = f(x)$, labelling the coordinates of all points where the graph intersects the coordinate axes.

[3]

a) Use polynomial division to divide $f(x)$ by $(x+1)$.

$$\begin{array}{r|l} & x^2 - 6x + 9 \\ x+1 & x^3 - 5x^2 + 3x + 9 \\ & -(x^3 + x^2) \\ & \quad -6x^2 + 3x \\ & \quad -(-6x^2 - 6x) \\ & \qquad\quad 9x + 9 \\ & \qquad\quad 9x + 9 \\ & \qquad\qquad\quad 0 \end{array}$$

Now factorise the quadratic.
What 2 numbers multiply to give 9 and add to give -6?
-3 and -3

repeated roots!

$x^2 - 6x + 9$

$= (x-3)^2 \quad$ OR $\quad (x-3)(x-3)$

So $\quad x^3 - 5x^2 + 3x + 9 = \boxed{(x+1)(x-3)^2}$

17 (a) **(4 marks)**

(a) Given that $(x+1)$ is a factor of $f(x) = x^3 - 5x^2 + 3x + 9$, fully factorise $f(x)$.

$\boxed{f(x) = (x+1)(x-3)^2}$ [4]

(b) Sketch the graph of $y = f(x)$, labelling the coordinates of all points where the graph intersects the coordinate axes.

[3]

b) The x^3 coefficient is 1, so this is a positive cubic:

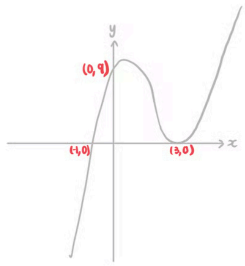
Roots occur when $f(x) = 0$

$(x+1)(x-3)^2 = 0$ repeated roots

$x = -1, \ 3, \ 3$

Find y-axis intercept by subbing $x = 0$ into eqn.

$y = (0+1)(0-3)^2 = (1)(-3)^2 = 9$



(b) **(3 marks)**

The vectors **a** and **b** are given by $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j}$ and $\mathbf{b} = -\mathbf{i} + 3\mathbf{j}$.

Find:

(i) $\mathbf{a} + \mathbf{b}$,
(ii) $5\mathbf{a}$,
(iii) $3\mathbf{a} - 2\mathbf{b}$,
(iv) $\mathbf{a} - t\mathbf{b}$.

[5]

UNDERLINE $\underline{i}$ AND $\underline{j}$ TO INDICATE VECTORS

i) $(3\underline{i} - 5\underline{j}) + (-\underline{i} + 3\underline{j})$

$\boxed{2\underline{i} - 2\underline{j}}$

ADD VECTORS LIKE ALGEBRA

ii) $5(3\underline{i} - 5\underline{j})$

$\boxed{15\underline{i} - 25\underline{j}}$

MULTIPLY VECTOR BY SCALAR

iii) $3(3\underline{i} - 5\underline{j}) - 2(-\underline{i} + 3\underline{j})$

TAKE CARE WITH NEGATIVES

$9\underline{i} - 15\underline{j} + 2\underline{i} - 6\underline{j}$

MULTIPLY OUT THEN COLLECT TOGETHER

$\boxed{11\underline{i} - 21\underline{j}}$

iv) $(3\underline{i} - 5\underline{j}) - t(-\underline{i} + 3\underline{j})$

TREAT t SAME AS NUMERICAL SCALAR

$3\underline{i} - 5\underline{j} + t\underline{i} - 3t\underline{j}$

FACTORISE TAKING OUT $\underline{i}$ $\underline{j}$ COMPONENTS

$\boxed{(3+t)\underline{i} - (5+3t)\underline{j}}$

TAKE OUT NEGATIVE AS FACTOR

18 **(5 marks)**

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle, where $\mathbf{F}_1 = 7\mathbf{i} - 2\mathbf{j}$ newtons and $\mathbf{F}_2 = -12\mathbf{i} - 10\mathbf{j}$ newtons.
The resultant force $\mathbf{R}$ acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$.

(a) Calculate the magnitude of $\mathbf{R}$ in newtons.

[3]

A third force $\mathbf{F}_3 = k\mathbf{j}$ newtons is to be applied to the particle. The constant k is to be selected so that the line of action of the new resultant force $\mathbf{R}_{\text{new}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ is at an angle of 45 degrees to the vector $\mathbf{j}$, measured anticlockwise.

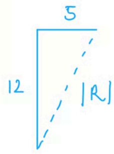
(b) Find the value of k.

[3]

a) $\underline{R} = \underline{F}_1 + \underline{F}_2 = 7\underline{i} - 2\underline{j} - 12\underline{i} - 10\underline{j}$

$= -5\underline{i} - 12\underline{j}$

$|\underline{R}| = \sqrt{5^2 + 12^2} = \boxed{13 \text{ newtons}}$



19 (a) **(3 marks)**

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle, where $\mathbf{F}_1 = 7\mathbf{i} - 2\mathbf{j}$ newtons and $\mathbf{F}_2 = -12\mathbf{i} - 10\mathbf{j}$ newtons.
The resultant force $\mathbf{R}$ acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$.

(a) Calculate the magnitude of $\mathbf{R}$ in newtons.

$\underline{R} = -5\underline{i} - 12\underline{j}$

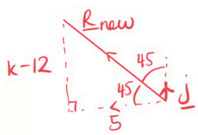
[3]

A third force $\mathbf{F}_3 = k\mathbf{j}$ newtons is to be applied to the particle. The constant k is to be selected so that the line of action of the new resultant force $\mathbf{R}_{\text{new}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ is at an angle of 45 degrees to the vector $\mathbf{j}$, measured anticlockwise.

(b) Find the value of k.

[3]

b)



$\underline{R}_{new} = \underline{R} + \underline{F}_3$

$= -5\underline{i} - 12\underline{j} + k\underline{j}$

$= -5\underline{i} + (k-12)\underline{j}$

For $\underline{R}_{new}$ to be at a $45°$ angle, the magnitudes of its $\underline{i}$ and $\underline{j}$ components must be equal.

$k - 12 = 5$

$\boxed{k = 17}$

(b) **(3 marks)**

In an experiment, three forces are acting on a particle. $\mathbf{F}_1 = 7\mathbf{i} - \mathbf{j}$ newtons and $\mathbf{F}_2 = x\mathbf{i} + y\mathbf{j}$ newtons are both constant forces, although the values of x and y are initially unknown. The third force is $\mathbf{F}_3 = k\mathbf{i} + k\sqrt{3}\mathbf{j}$ newtons, where $k \geq 0$ is a parameter that can be varied by the experimenters. The resultant force $\mathbf{R}$ acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$.

(a) Given that $\mathbf{R} = \mathbf{0}$ when the magnitude of $\mathbf{F}_3$ is 10 newtons, find the exact values of x and y.

[4]

(b) Find the magnitude of $\mathbf{F}_2$ and the angle it makes with the vector $\mathbf{i}$. Give your answers correct to 1 decimal place.

[3]

a) $|F_3| = \sqrt{k^2 + (k\sqrt{3})^2} = 10 = 2k$

$k = 5$

$\underline{F}_3 = 5\underline{i} + 5\sqrt{3}\underline{j}$

$\underline{R} = 7\underline{i} - \underline{j} + x\underline{i} + y\underline{j} + 5\underline{i} + 5\sqrt{3}\underline{j}$

$= 0\underline{i} + 0\underline{j}$

$7 + x + 5 = 0$

$\boxed{x = -12}$

$-1 + y + 5\sqrt{3} = 0$

$\boxed{y = 1 - 5\sqrt{3}}$

20 (a) **(4 marks)**

In an experiment, three forces are acting on a particle. $\mathbf{F}_1 = 7\mathbf{i} - \mathbf{j}$ newtons and $\mathbf{F}_2 = x\mathbf{i} + y\mathbf{j}$ newtons are both constant forces, although the values of x and y are initially unknown. The third force is $\mathbf{F}_3 = k\mathbf{i} + k\sqrt{3}\mathbf{j}$ newtons, where $k \geq 0$ is a parameter that can be varied by the experimenters. The resultant force $\mathbf{R}$ acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$.

(a) Given that $\mathbf{R} = \mathbf{0}$ when the magnitude of $\mathbf{F}_3$ is 10 newtons, find the exact values of x and y.

$x = -12$ $\quad$ $y = 1 - 5\sqrt{3}$ $\quad -7.66\ldots$

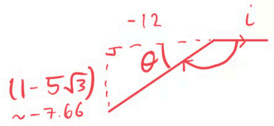
[4]

(b) Find the magnitude of $\mathbf{F}_2$ and the angle it makes with the vector $\mathbf{i}$. Give your answers correct to 1 decimal place.

[3]

b) $|\underline{F}_2| = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + (1 - 5\sqrt{3})^2}$

$= \boxed{14.2 \text{ Newtons (1dp)}}$



$\alpha = 180° - \theta$

$= 180 - \tan^{-1}\left(\frac{7.66\ldots}{12}\right) = \boxed{147.4° \text{ (1dp) clockwise from } \underline{i}}$

(b) **(3 marks)**

End of year

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6. Find the length of the side PQ in the triangle PQR below, giv...
7. Find the coordinates of the midpoint of the straight line co...
8. In triangle A B C comma space space stack A B with rightwa...
9. Show that the equation 4 x plus 2 y minus 6 equals 0 can b...
10. Find the length of the straight line segments connecting th...
11. On separate diagrams sketch the circles with the following...
12. Complete the square for (i) x squared plus 8 x minus 4 (ii) ...
13. Expand and simplify (i) left parenthesis 2 x plus 3 right par...
14. Solve the inequalities: (i) 2 x greater or equal than 8 (ii) 3 p...
15. Use the factor theorem to verify that left parenthesis x mi...
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18. The vectors a and b are given by bold a equals 3 bold i mi...
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